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Resonance Production of Three Neutral Supersymmetric Higgs Bosons at LHC

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Abstract

Multiple production of Higgs particles is essential to study Higgs self-couplings at future high–energy colliders. In this paper we calculated the resonance contributions to the production of three lightest neutral supersymmetric Higgs bosons in gluon fusion at LHC. The cross sections due to trilinear Higgs couplings is sizeable but the measurement of the quartic coupling $\lambda_{hhhH(h)}$ seems to be impossible.

1 Introduction

One of the basic open questions of particle physics is the nature of electroweak symmetry breaking. Beyond the discovery of the Higgs boson(s), however, reconstructing the Higgs potential will be necessary, and that requires the experimental study of the self–couplings of the Higgs bosons.

Pair production of neutral Higgs particles in gluon fusion sensitive to the trilinear couplings of Higgs bosons was studied in the SM [1] and the Minimal Supersymmetric Standard Model (MSSM) [2–5]. QCD corrections were included in the limiting case of a heavy top mass [6].

In the SM the trilinear and the quartic couplings of the physical Higgs particle are fixed by the Higgs mass. There is no resonance in the pair production of SM Higgses, and the cross section is only $\simeq$20–50 fb in the intermediate mass range [3]. No resonance effect is present in the production of three Standard Model Higgs bosons via one Higgs in gluon fusion and it is estimated to be under the discovery limit at LHC.

In the MSSM there are five Higgs bosons $(h, H, A, H^{\pm})$ and many trilinear and quartic couplings among them. Two parameters describe the Higgs sector of the MSSM: M_A, the mass of the CP-odd Higgs boson A, and tanβ, the ratio of the two vacuum expectation values. For a wide range of M_A, in pp collision only the cross section of the hh production is large [2,3]. Possible other processes, such as the gauge boson fusion and Higgs–strahlung off W bosons or heavy quarks (top) provide less event number.

In order to provide further tests of trilinear and quartic Higgs self–couplings, in this paper we calculate the resonance enhanced production of three lightest supersymmetric Higgs bosons (h) in gluon fusion

$$pp \to gg \to hhh. \tag{1}$$

Generally four different graphs contribute to the production of three Higgses. Gluons couple to triangle, box or pentagon quark loops emitting 1,2 or 3 Higgs bosons, as is seen in Fig.1. Squark loops are neglected. We have omitted the pentagon graph producing a flat continuum background for the resonance production and we will give the complete cross section valid in a wider range in a subsequent publication [7]. We learn that the quartic coupling is not accessible by LHC experiment. The resonance contribution to (1), however, sizeable at LHC For instance at $\tan\beta = 3$ it yields a cross section of about 300 fb. This is about 1/10 times the corresponding hh–case [3].

The masses, widths and the couplings were calculated using the complete one–loop and the leading two–loop radiative corrections from [8]. The relevant Higgs self–couplings are the following

$$\begin{aligned}
\lambda_{hhh} \quad &= 3\cos(2\alpha)\sin(\beta+\alpha) + \frac{3\epsilon}{M_Z^2}\frac{\cos^3\alpha}{\sin\beta} \\
\lambda_{Hhh} \quad &= 2\sin(2\alpha)\sin(\beta+\alpha) - \cos(2\alpha)\cos(\beta+\alpha) + \frac{3\epsilon}{M_Z^2}\frac{\sin\alpha\cos^2\alpha}{\sin\beta}
\end{aligned}$$

$$
\begin{aligned}
\lambda_{HHh} &= -2\sin(2\alpha)\cos(\beta+\alpha) - \cos(2\alpha)\sin(\beta+\alpha) + \frac{3\epsilon}{M_Z^2}\frac{\sin^2\alpha\cos\alpha}{\sin\beta} \\
\lambda_{Hhhh} &= 3\sin(2\alpha)\cos(2\alpha) + \frac{\epsilon}{8M_Z^2}\frac{\cos\alpha\sin^2\alpha}{\sin^2\beta} \\
\lambda_{hhhh} &= 3\cos^2(2\alpha) + \frac{\epsilon}{4M_Z^2}\frac{\sin^4\alpha}{\sin^2\beta}
\end{aligned}
\tag{2}
$$

The trilinear and quartic couplings are normalized to $\lambda_3 = [\sqrt{2}G_F]^{1/2}M_Z^2$ and $\lambda_4 = \sqrt{2}G_F M_Z^2$, respectively. The couplings depend on β and the mixing angle α of the CP–even Higgs sector

$$
\tan 2\alpha = \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2 + \epsilon/\cos 2\beta}\ \tan 2\beta. \tag{3}
$$

The leading m_t^4 one–loop corrections [9] are parametrized by

$$
\epsilon = \frac{3G_F}{\sqrt{2}\pi^2}\frac{m_t^4}{\sin^2\beta}\ \log\left[1 + \frac{M_S^2}{m_t^2}\right]. \tag{4}
$$

The common squark mass is fixed to M_S= 1 TeV. The quartic couplings depend on M_A and are shown for two values of tanβ = 3 and 30 in Fig.2.

The paper is organized as follows. In the next section we give the cross section of the resonance production of three h's in gluon fusion and in section 3 we show the numerical results.

2 The cross section

Three different channels contribute to the resonance production of three lightest supersymmetric Higgs particles (h) in gluon fusion. (a) One virtual Higgs boson (h,H) is produced by the heavy quark triangle and it decays into 3 h's via the quartic coupling λ_{Hhhh} or λ_{hhhh}, Fig. 1a. (b) One virtual Higgs boson decays into 3 h's in two steps testing trilinear couplings, $gg \to H, h \to (H,h)h \to hhh$, Fig. 2b. (c) Heavy quark box diagram couples to two Higgses, one of them decays subsequently into two h's, $gg \to (H,h)h \to hhh$, Fig. 2c. We have omitted the pentagon graphs contributing only to the continuum production. We believe it increases the production at large tanβ due to the large hbb–coupling, but does not change our conclusion about the measureability of the quartic Higgs–couplings.

There are two different helicity amplitudes contributing to the total cross section: the total spin of the two gluons along the collision axis can be $S_z = 0$ (contribution F) or $S_z = 2$ (contribution G). The triangle graphs give only contributions F_3 ($S_z = 0$) as they involve a single spin–0 Higgs intermediate state. Box graphs give both contributions F_4 ($S_z = 0$) and G_4 ($S_z = 2$). F_4 and G_4 are Lorentz and gauge invariant decompositions of the box amplitude. The tensor basis is the following

$$
S_z = 0 \quad A^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{(p_1 p_2)} \tag{5}
$$

$$S_z = 2 \quad B^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2(p_1p_2)}(p_3^2 p_1^\nu p_2^\mu - 2(p_2p_3)p_1^\nu p_3^\mu - 2(p_1p_3)p_2^\mu p_3^\nu + 2p_3^\mu p_3^\nu(p_1p2))$$

where p_1, p_2 are the momenta of the incoming gluons and p_3 is one of the momenta of the outgoing Higgs bosons (p_3, p_4, p_5). Here $p_T^2 = 2\frac{(p_1p_3)(p_2p_3)}{(p_1p_2)} - p_3^2$ is the transverse momentum of the third particle. Tensors $A^{\mu\nu}$ and $B^{\mu\nu}$ are orthogonal and normalized to 2 [2].

The M–matrix of the process is

$$\mathcal{M} = \frac{(\sqrt{2}G_F)^{3/2}\alpha_s\hat{s}}{4\pi}\ \epsilon_a^\mu \epsilon_b^\nu\ \delta_{ab}(FA_{\mu\nu} + GB_{\mu\nu}). \tag{6}$$

The spin and color averaged parton level cross section of the process $gg \to hhh$ is

$$d\hat{\sigma}(gg \to hhh) = \frac{\sqrt{2}G_F^3\alpha_s^2}{1024(2\pi)^6}\Big[|F|^2 + |G|^2\Big]\frac{\lambda^{1/2}(\hat{s}_{45}, p_4^2, p_5^2)}{\hat{s}_{45}}d\hat{s}_{45}d\hat{s}_{13}d\Omega_{45}^{CM}. \tag{7}$$

Here we used the Chew-Low parametrization of the three particle phase space [10], a trivial angle integration was carried out, $\hat{s}_{ik} = (p_i + p_k)^2$, and the usual lambda function is $\lambda(x, y, z) = \Big(x - (\sqrt{y} + \sqrt{z})^2)(x - (\sqrt{y} - \sqrt{z})^2\Big)$. $d\Omega_{45}^{CM}$ is the differential solid angle in the center of mass system of particles 4 and 5.

F receives contributions from the triangle and box graphs, while only box diagrams give contributions $S_z = 2$ to G

$$\begin{aligned} F &= \sum_{t,b}(C_3\ F_3\ + \sum_{i=3,4,5} C_4^{(i)}\ F_4^{(i)}) \\ G &= \sum_{t,b}\sum_{i=3,4,5} C_4^{(i)} G_4^{(i)}. \end{aligned} \tag{8}$$

Here we separated the functions F_3, F_4, G_4 responsible for the heavy quark triangle and boxes, C_3, C_4 contain the coupling constants and the propagators. The amplitude has to be summed over all quark flavours. However, the four light quarks can be neglected having very small Higgs couplings. Supersymmetric particles are assumed to be too heavy to participate in the triangle or box loop. The second sum in the box contribution corresponds to the outgoing Higgs particle that couples directly to the quark loop.

We calculated F_3, F_4, G_4 and found them in agreement with the results of [2], [1]. For instance,

$$F_3 = 2\frac{m_Q^2}{\hat{s}}\Big(2 + (4m_Q^2 - \hat{s})C_{12}\Big), \quad \hat{s} = \hat{s}_{12}. \tag{9}$$

where

$$C_{ij} = \int \frac{d^4q}{i\pi^2}\ \frac{1}{(q^2 - m_Q^2)\Big[(q + p_i)^2 - m_Q^2\Big]\Big[(q + p_i + p_j)^2 - m_Q^2\Big]}. \tag{10}$$

F_4, G_4 have long analytic expressions which can be found in ref [2]. The box functions $F_4^{(i)}, G_4^{(i)}$ in (8) depend on three momenta (p_1, p_2, p_i), $\quad i = 3, 4, 5$ and $F_4^{(i)} = F_4(p_1, p_2, p_i)$ etc.

In case of a large quark mass ($m_Q^2 \gg \hat{s} \sim M_{h,H}^2$) we get as in [2]

$$F_3 = \frac{2}{3} + \mathcal{O}\left(\hat{s}/m_Q^2\right), \qquad F_4^{(i)} = -\frac{2}{3} \quad \mathcal{O}\left(\hat{s}/m_Q^2\right), \quad G_4^{(i)} = \mathcal{O}\left(\hat{s}/m_Q^2\right) \tag{11}$$

In the limit of light quark masses ($m_Q^2 \ll \hat{s} \sim M_{h,H}^2$) all the form factors vanish to $\mathcal{O}\left(m_Q^2/\hat{s}\right)$.

The generalized triangle couplings are the following

$$\begin{aligned} &C_3 = C_3^h + C_3^H + C_3^{hh} + C_3^{hH} + C_3^{Hh} + C_3^{HH} \\ &C_3^{H_a} = \lambda_{hhhH_a} \frac{M_Z^2}{\hat{s} - M_{H_a}^2 + iM_{H_a}\Gamma_{H_a}} g_Q^{H_a} \\ &C^{H_aH_b} = \lambda_{H_aH_bh} \frac{M_Z^2}{\hat{s} - M_{H_a}^2 + iM_{H_a}\Gamma_{H_a}} \lambda_{H_bhh} \frac{M_Z^2}{\hat{s} - M_{H_b}^2 + iM_{H_b}\Gamma_{H_b}} g_Q^{H_a}, \quad H_{a,b} = h/H. \end{aligned} \tag{12}$$

The box couplings are

$$\begin{aligned} &C_4 = C_4^h + C_4^H \\ &C_4^{(h,H)(i)} = \lambda_{hh(h/H)} \frac{M_Z^2}{\hat{s}_{kl} - M_{h/H}^2 + iM_{h/H}\Gamma_{h/H}} g_Q^h g_Q^{h/H}, \end{aligned} \tag{13}$$

i, k, l are cyclic. $g_Q^{h/H}$ denotes the Higgs–quark couplings normalized to the SM Yukawa coupling $[\sqrt{2}G_F]^{1/2}m_Q$,

$$g_t^h = \cos\alpha/\sin\beta, \quad g_b^h = -\sin\alpha/\cos\beta, \quad g_t^H = \sin\alpha/\sin\beta, \quad g_b^H = \cos\alpha/\cos\beta. \tag{14}$$

3 Results

Now, we calculate the production cross section of three lightest neutral supersymmetric Higgs bosons $pp \to hhh + X$ via gluon fusion at the LHC energy of $\sqrt{s} = 14$ TeV in the above picture by folding the parton level fusion cross section with the gluon luminosity. We used the GRV structure functions [11] for the gluon luminosity at $Q^2 = \hat{s}$. The numerical integration was made by the VEGAS package [12]. The total cross section vs. M_A is shown in Fig.3 for two represantative values of tan$\beta = 3$ and tan$\beta = 30$. In the plotted range of M_A the heavy CP–even Higgs boson (H) is nearly degenerate in mass with A while the mass of the lightest neutral Higgs h is quickly approaches approximately 104 GeV from below.

For tan$\beta = 3$ in the range of M_A = 200–350 GeV the cross section is enhanced by resonance effect and large. The cross section goes up to nearly 300 fb and slowly decreases to about 150 fb giving a large number of events at the expected luminosity $\int Ldt \simeq 100 fb^{-1}/year$. The main contribution comes from the box diagram where the heavy CP–even Higgs boson H couples to the quark loop and decays into two bosons h. M_h slowly increases around 100 GeV when $M_H \simeq M_A$ reaches 200 GeV the H propagator

enters the resonance region. At $M_H \sim 2m_t$ there is a threshold effect due to the fall–off of the branching ratio $BR(H \to hh)$ and partly because on–shell top quark pairs can be produced in the quark loop. For large values of M_A the cross section reaches a continuum value of a few fb. The dip at small M_A, similarly to the case of tan$\beta = 30$, is the result of the zero in the trilinear couplings $\lambda_{(H/h)hh}$.

For tanβ=30 the main contribution comes from the box diagram where an h couples to the quark loop, propagates and decays into two h's via λ_{hhh}. The Hhh coupling is small compared to the case of tan$\beta = 3$ while the hhh coupling is larger giving a sizeable continuum. There is no observable resonance effect and the cross section is nearly constant, 6 fb, versus M_A after the coupling constant changed sign. The K factor is expected to increase the cross section as in other cases [6,3].

The contributions of the diagrams containing a heavy quark triangle are also shown in Fig.3 at tanβ=3. The lowest curve corresponds to the diagram with quartic couplings, the one in the middle sums up all the contributions of the graphs involving a quark triangle. There is a clear resonance effect in both curves when M_H reaches $3M_h$ at ~310 GeV. Here the first propagator of $gg \to H \to Hh \to hhh$ becomes resonant too. The first rise in the middle curve is the result of the resonance in the second propagator of Fig. 2b, similarly to the box. The triangle loops yield, however, only a small fraction of the cross section even at their peak at 300-350 GeV.

The suppressed contribution of the quartic coupling implies that in $pp \to hhh + X$ the measurement of the quartic coupling is not possible. Not only the box contribution is larger by two orders of magnitude but also the other triangle diagrams are 10 times larger. For tan$\beta = 30$ the case is even worse.

In conclusion, in this paper we have calculated the resonance contribution to the cross section of 3h–production at LHC via gluon fusion. The main contribution comes from the trilinear Higgs coupling while the quartic ones turn out to be negligible. In the resonance region at small tanβ the ratio of 2h and 3h production is about 10.

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FIGURE CAPTIONS

Fig.1: Generic diagrams contributing to the production of three CP–even MSSM Higgs bosons in gluon–gluon collisions, $gg \to hhh$: a) triangle with quartic coupling, b)triangle with trilinear couplings c) box , d) pentagon contributions.

Fig.2: Quartic Higgs couplings in the MSSM as functions of the pseudoscalar Higgs mass M_A for two representative values of tanβ=3 and 30.

Fig.3: Total cross sections for production of three lightest CP–even MSSM Higgs bosons in gluon–gluon collisions at the LHC for tan$\beta = 3$ and 30 (upper two curves). The upper axis presents the scalar Higgs masses M_h for tan$\beta = 3$ corresponding to the pseudoscalar masses M_A. The lower two dashed curves represent the resonance contributions of the triangle graphs of Fig. 2a and 2b for tanβ=3.

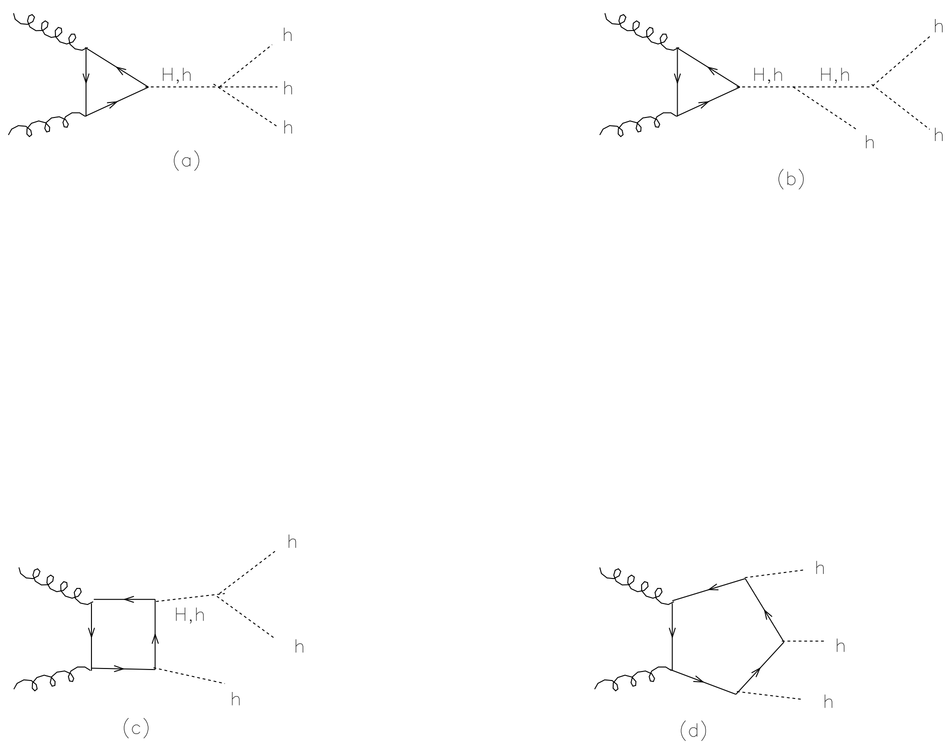
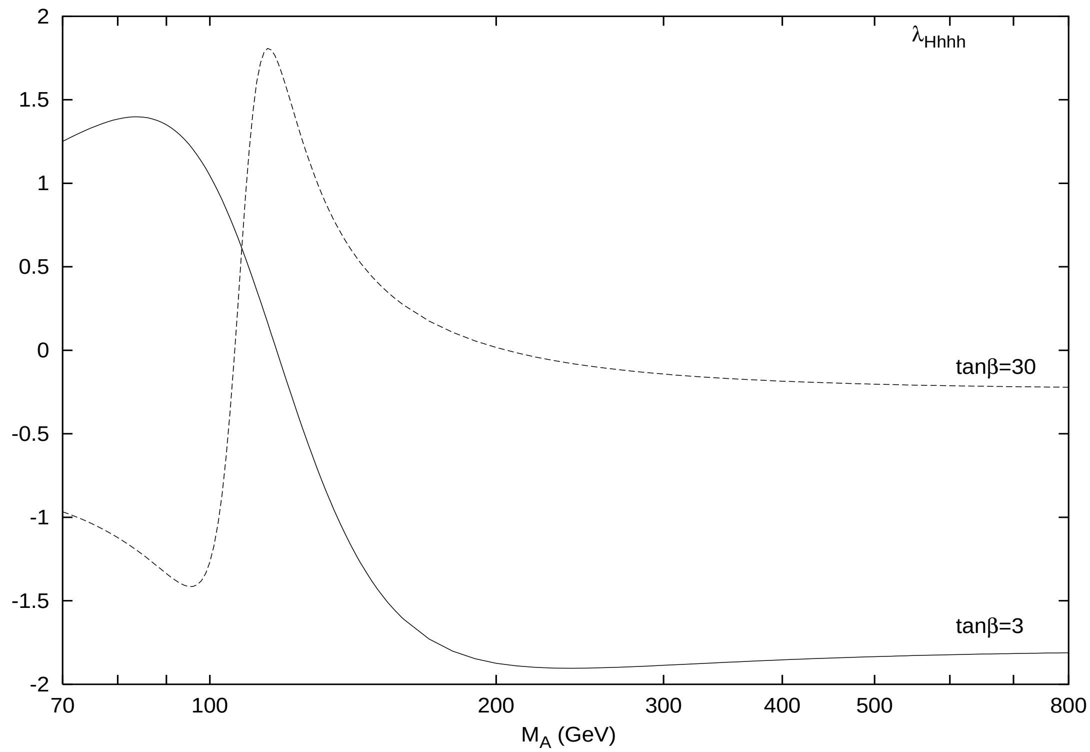
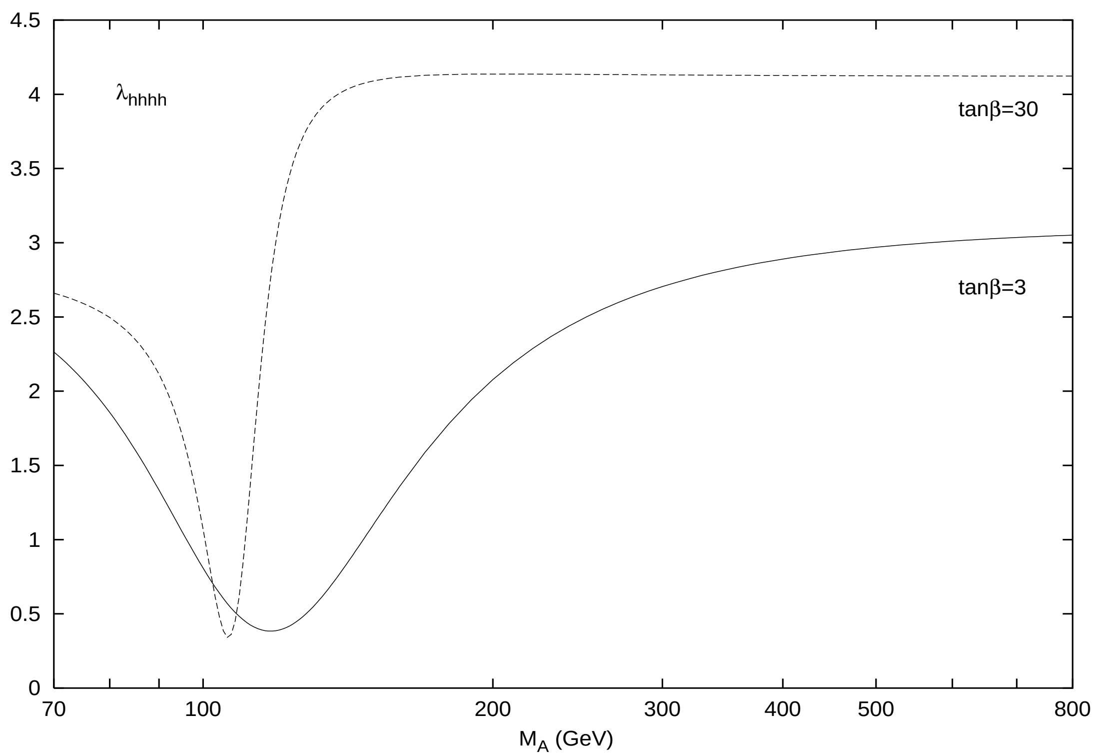


Fig. 1



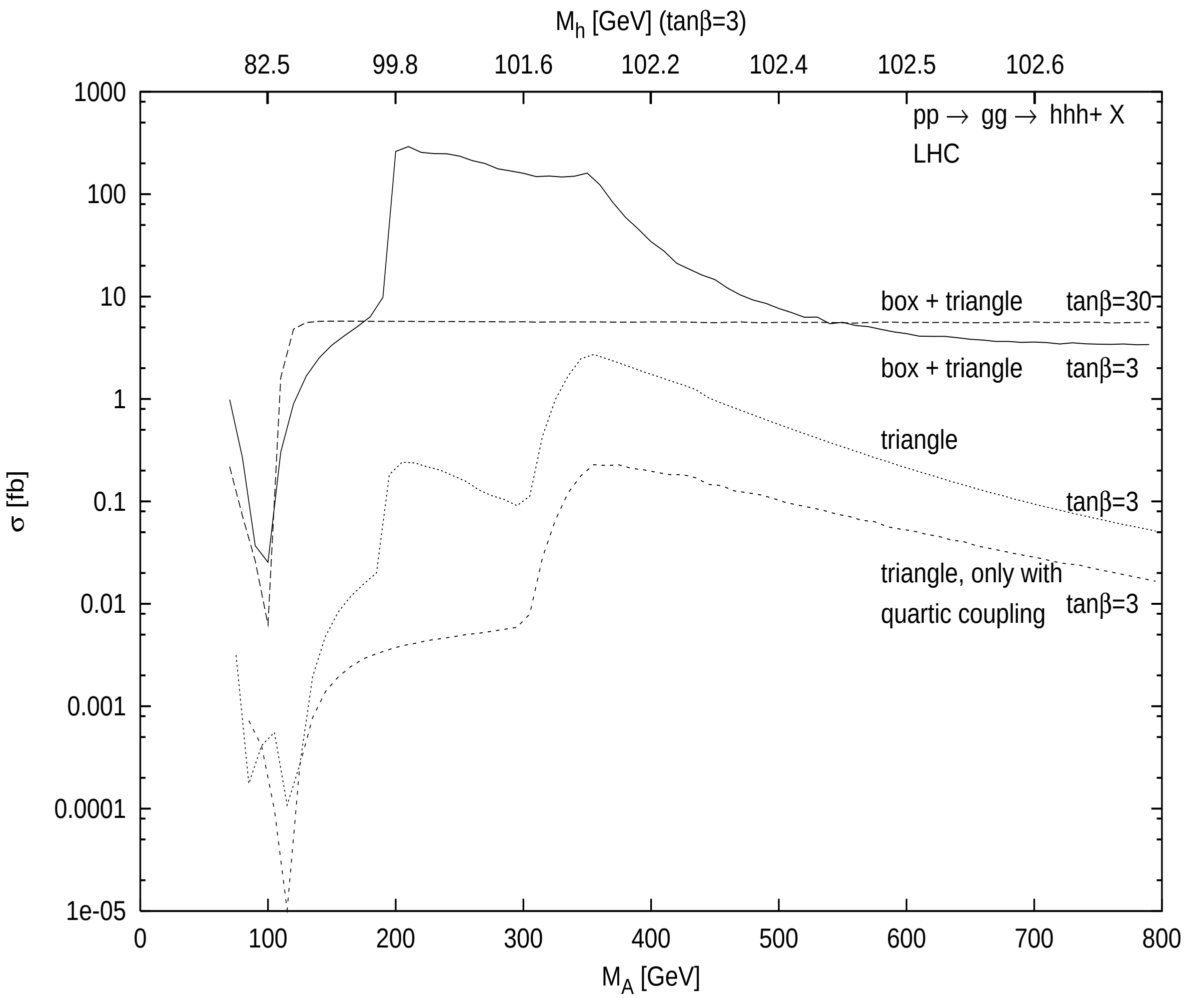
λ_{Hhhh}
tanβ=30
tanβ=3
M_A (GeV)

Fig. 2a



λ_{hhhh}
tanβ=30
tanβ=3
M_A (GeV)

Fig. 2b



M_h [GeV] (tanβ=3)
82.5
99.8
101.6
102.2
102.4
102.5
102.6
1000
100
10
1
0.1
0.01
0.001
0.0001
1e-05
σ [fb]
pp → gg → hhh+ X
LHC
box + triangle
tanβ=30
box + triangle
tanβ=3
triangle
tanβ=3
triangle, only with
quartic coupling
tanβ=3
0
100
200
300
400
500
600
700
800
M_A [GeV]

Fig. 3